The Cuprate Pseudogap:
Competing Order Parameters or Precursor Superconductivity

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In this paper we compare two broad classes of theories for the pseudogap in cuprate superconductors. The comparison in made in reference to measurements of the superfluid density, $\rho_s(T,x)$, in $YBa_2CuO_{7-\delta}$ films having a wide range of stoichiometries, δ, or, hole doping, x. The theoretical challenge raised by these (and previous) data is to understand why the T-dependence of $\rho_s(T,x)$ is insensitive to the fermionic excitation gap $\Delta(T,x)$, which opens in the normal state and persists into the superconducting state, when presumably $\rho_s(T)$ is governed, at least in part, by fermionic excitations. Indeed, $\rho_s(T,x)$ seems to have a BCS-like dependence on $T_c(x)$, which, although not unexpected, is not straightforward to understand in pseudogapped superconductors where $T_c(x)$ and the excitation gap have little in common. Here, we contrast "extrinsic" and "intrinsic" theoretical approaches to the pseudogap and argue that the former (for example, associated with a competing order parameter) exhibits more obvious departures from BCS-like T dependences in $\rho_s(T)$ than approaches which associate the pseudogap with the superconductivity itself. Examples of the latter are Fermi liquid based schemes as well as a pair fluctuation mean field theory. Thus far, the measured behavior of the superfluid density appears to argue against an extrinsic interpretation of the pseudogap, and supports instead its intrinsic origin.

One of the most interesting aspects of high temperature superconductivity in hole-doped cuprates is its development out of a non-Fermi-liquid normal state that has a fermionic (pseudo)gap. [1] This gap, $\Delta(T)$, is present at the onset of superconductivity and persists into the superconducting state. Understanding this interplay of superconductivity and the pseudogap is the goal of the present paper. We argue here that the most natural way to proceed is to study a property associated with the superconducting phase only: the superfluid density $\rho_s(T)$. We do so here in the context of two different classes of theories of the cuprate pseudogap, extrinsic and intrinsic theories, in which the normal-state gap is extrinsic to superconductivity or is a precursor, respectively. These theories are compared with systematic measurements of $\rho_s(T,x)$, in films of $YBa_2Cu_3O_{7-\delta}$ with various hole doping concentrations, x. The unusual aspect of the data is that they appear to follow a universal BCS-like scaling with $T_c(x)$. This behavior is unusual because the pseudogap is present above T_c and persists below T_c, and therefore must be included in the analysis. It appears at the outset that any theory that includes the pseudogap physics will predict nonuniversal behavior since the pseudogap grows as x decreases, while the T_c decreases.

Experiments show that the pseudogap persists below T_c. This persistence is to be understood as implying that the underlying "normal" phase of the superconductor is distinctly different from the simple Fermi liquid phase that underlies strict BCS theory. Loram and Tallon [1] have argued phenomenologically for a persistent pseudogap on the basis of thermodynamic data. In order to obtain a physical meaningful entropy for the extrapolated normal state, they posit that a pseudogap is present at all $T \leq T_c$ as an additional term, Δ_{pg}, in the fermionic dispersion. This analysis forms the basis for their inferred condensation energies. Similarly, tunneling measurements [2,3], particularly the earliest STM mea-

surements from Fischer and co-workers [4], have led to the conclusion that the $T \leq T_c$ normal state (e.g., measured inside a vortex core) contains a pseudogap, rather similar to that of the normal state.

One category of theory of this persistent pseudogap has the pseudogap arising from physics *extrinsic* to the superconductivity, e.g., deriving from a bandstructure effect [5–7] or from a competing order parameter [8,9]. This extrinsic pseudogap approach has been formulated principally at a mean field level, which can be reasonably well justified on the basis of the fact that experiments seem to indicate only a narrow critical regime [10] and in some (but not all) respects these pseudogapped cuprates are surprisingly similar to conventional mean field (*i.e.,* BCS) predictions, as demonstrated below.

Another category of pseudogap theory, which we refer to as *intrinsic*, has the pseudogap arising from superconductivity itself. Among these are the phase fluctuation scenario [11], Fermi-liquid based superconductivity along the lines of a phenomenology set forward by Lee and Wen [12,13], stripe-based theories [14] as well as pair fluctuation approaches [15,16].

We will argue that the superfluid density provides a suitable basis for comparing and contrasting these different theories. Here we address extrinsic approaches with the D-Density wave (DDW) theory as a prototype, as well as two types of intrinsic schemes: Fermi liquid based superconductivity and a pair fluctuation theory. At the mean field level [16], the latter will be seen to be in the same spirit as the the DDW mean field approach.

Within the intrinsic school, calculations of ρ_s are frequently associated with boson-fermion models. Among the first such studies was the work of Stroud and co-workers [17] and of Carlson *et al* [18] which pointed out that the T-linear decrease of $\rho_s(T)$ might be due at least in part to (bosonic) phase fluctuations, rather than nodal quasiparticles in a d-wave superconductor. Subsequent work [14] introduced *d-*

wave fermionic excitations. Ioffe and Millis [19] have argued that the Fermi liquid-based phenomenology of Lee and Wen can be interpreted in terms of coupling between phase fluctuations and nodal quasi-particles. Orenstein and co-workers [20] have studied the effects of (bosonic) collective phase modes on ρ_s in the context of an inhomogeneous model for superconductivity designed, however, for the overdoped phase.

Finally, the pair-fluctuation approach to the pseudogap [16] provides yet another boson-fermion model. This scheme is closely related to Hartree-approximated Ginsburg-Landau (GL) theory [21], where one sees how T_c is suppressed by beyond-Gaussian fluctuations or non-condensed pairs, which give rise simultaneously to a fermionic excitation gap in the normal state and, thus, a breakdown of Fermi liquid theory [22]. In this picture, non-condensed bosons above and below T_c, and a fermionic pseudogap, are two sides of the same coin [23]. As a consequence it follows that the excitation gap and the order parameter are different above and (at least for some range of temperatures) below T_c, as well. For this reason $\rho_s(T)$ is necessarily affected by these bosonic degrees of freedom.

There are increasingly reports of new broken symmetries as well as arguments for quantum critical points associated with the pseudogap phase [9,24] which support an extrinsic approach to the pseudogap. At the same time, observations of Nernst signals [25] above T_c which evolve continuously into the vortex contributions below T_c, and the smooth evolution through T_c of the excitation gap, provide support for a scenario in which the pseudogap is *intrinsic* to the superconductivity. Thus, there is a substantial need to compare the two schools directly and here we do this below T_c where the predictions are most clear cut.

A. Experimental Data and Constraints Imposed

There is a strong consensus in the field that fermionic d-wave quasiparticles dominate thermodynamics and transport of the superconducting state, e.g., the T-linear decrease of $\rho_s(T)$ at low T is due to thermally excited BCS-like quasiparticles near the nodes in the gap, Δ. This view is natural for overdoped cuprates that do not exhibit a pseudogap above T_c. To extend it to underdoped cuprates, in which a d-wave gap $\Delta(T)$ turns on at a temperature T^* substantially above T_c, and is relatively constant from T_c down to $T = 0$, theory must account for the pseudogap in the superconducting state. It is convenient to characterize the strength of the pseudogap below T_c by an experimental parameter: $\alpha = \Delta(T_c)/\Delta(0)$. When $\alpha \approx 0$ the system is "BCS-like". α is of order unity in the strong pseudogap regime.

For pedagogical purposes, in Figure 1, we present a schematic plot of the temperature dependence of the excitation gap within the superconducting regime, for a range of hole concentrations ranging from very underdoped where $\Delta(T)$ is essentially a constant, to overdoped, where $\Delta(T)$ follows the BCS T dependence. This plot, which can be viewed as a consolidation of an analysis, such as that presented in

Ref. [1], indicates the role played by the important parameter α. This simple figure in conjunction with experimental data
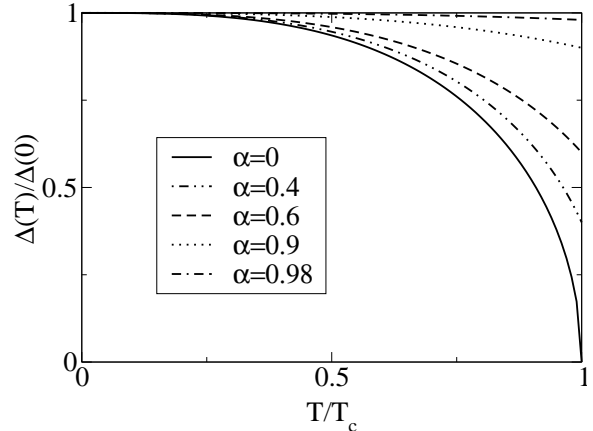

FIG. 1. Schematic plot of the temperature dependence of the excitation gaps below T_c, for different hole concentrations.

on $\rho_s(T)$ to be presented below provides a restatement of a central theme in this paper: that, while the T dependence of the excitation gap varies dramatically as the system becomes progressively more underdoped, nevertheless one sees rather little change in the temperature dependence of the superfluid density.

We make two more important points about T_c and T^*. We note the experimental observation [26,27] that the zero temperature gap $\Delta(0, x)$ is proportional to the onset temperature $T^*(x)$:

$$2\Delta(0, x)/(k_B T^*) \approx 2.15 \qquad (1)$$

which, because it satisfies the BCS condition, supports the idea that the pseudogap and superconductivity have a common origin. [27] If so, then the challenge is to understand why ρ_s vanishes so far below T^*. Another central equation which is reasonably well satisfied by the data presented below, is known as the Uemura constraint [28]

$$T_c(x) = \nu \rho_s(0, x) \qquad (2)$$

Here ν is an unimportant constant. Presumably this equation is intimately connected to the physics associated with a Mott insulator.

Measurements of the in-plane superfluid density in $YBa_2Cu_3O_{7-\delta}$ films at various δ, upper inset to Fig. 2, have rather similar shapes. Films were grown by co-evaporation and consistently showed a linear low-T penetration depth and inductive transition less that $0.5K$ wide as grown. Deoxygenation of the films was accomplished by heat treating the films in an Ar atmosphere at $250°C$ for ten minute intervals. The sheet conductivity, $\sigma = \sigma_1 - i\sigma_2$, was determined from the mutual inductance of coaxial coils driven at 50 kHz located on opposite sides of the film. σ_1 is much smaller than σ_2 everywhere except close to T_c. From σ_2 we define $\lambda^{-2} = \mu_0\sigma_2\omega/d$, where d is the film thickness. For purposes of later comparison with calculations, we define $\rho_s \propto \lambda^{-2}$ to have units of
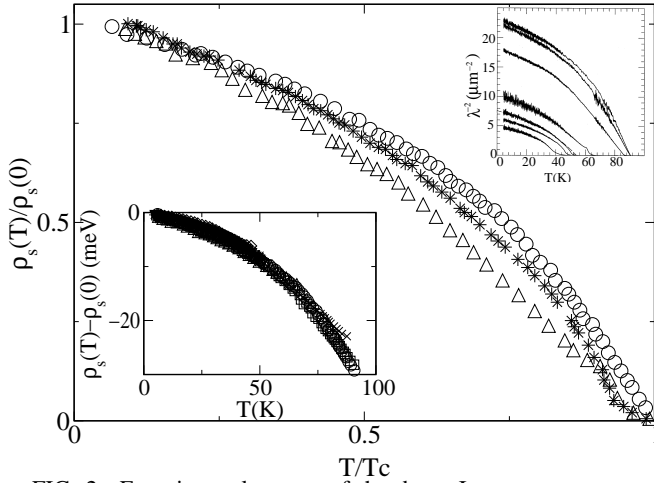
FIG. 2. Experimental survey of the data. Inverse square penetration depth for YBCO films with varying doping plotted in three different ways.

energy: $\rho_s \equiv \hbar^2 d_c \lambda^{-2}/2e^2\mu_0$. d_c is the c-axis lattice constant in YBCO, about 1.17 nm.

The similarity of behavior for different oxygen concentrations can be seen either by plotting normalized data, of which three typical curves are shown in the main portion of Figure 2, or by plotting the change in superfluid density, $\rho_s(T) - \rho_s(0)$ vs. T, lower inset. In brief, the quasiuniversal aspects of the data are that the low-T slope changes little with underdoping, and the curvature at low T remains negative and small. These results are consistent with earlier experiments [29] on LaSrCuO as well as with single crystal YBaCuO data at two different oxygen stoichiometries [30]. Results such as these which show little difference between the overdoped (or optimal) samples where BCS theory is expected to apply, and underdoped samples, are the principal reason for the widespread belief that below T_c pseudogap effects disappear, and the material becomes an ordinary BCS superconductor. The only energy scale evident in the data is T_c, in spite of the growing pseudogap with underdoping.

Finally, we note that data on films do not show evidence for a significant critical region near T_c for any x. The underdoped films exhibit some inhomogeneity near T_c, so the experimental case for them is not strong, but the transition in the optimally doped film is less than 0.5 K wide, and for it the case is strong. In the end, we assume that it is reasonable to examine the data in the light of theories that are not too far from mean-field. It should also be noted that the universality in $\rho_s(T)$ is evident at *all* temperatures. This is most apparent in the off-set plot, but this and the rescaled plot suggest that the behavior around T_c is as important, and needs to be understood as systematically as the behavior around $T = 0$. On the basis of this universality, one might also conclude that some form of mean field theory which does not stray too far from the BCS picture is a reasonable starting point for addressing these $\rho_s(T)$ data. This viewpoint provides the background for some of the theoretical discussion which follows.

B. Fermi Liquid Based Superconductivity

In the inset of Figure 3 we present predictions for $\lambda^{-2}(T)$ based on BCS theory, given that T^* is the transition temperature, and given the experimental [1] behavior of the magnitude of the excitation gap. The solid lines labelled "BCS" plot the BCS predictions for $\lambda^{-2}(T) \propto \rho_s^{BCS}(\Delta(T))$, for three values of x, with indicated values of the parameter α, introduced above. Here the momentum dependence of the d-wave gap was chosen to nearly fit the slope in the optimal case ($\alpha = 0.4$). The same gap shape was then used for all x. The bandwidth $t(x)$ was chosen to agree with the measured $\lambda^{-2}(x, 0)$.

This disagreement between experiment and BCS theory, as applied in this plot, might be explained within three different scenarios. In the extrinsic pseudogap model the BCS curves are inappropriate since T^* and the normal state excitation gap have nothing to do with superconductivity. Alternative Fermi liquid based theories of the superconducting state [12,13] belong to the intrinsic school which presumes that the pseudogap derives from superconductivity. They introduce Landau parameters to fit the low T slope. Finally, one might argue that there are other forms of excitation of the condensate (i.e., bosonic pair fluctuations) which form the basis for a mean field theoretic intrinsic approach [16] to be discussed later.

In BCS d-wave theory with a tight-binding band model with nearest-neighbor hopping, t, the low-T behavior of ρ_s is given by:

$$[\rho_s(T) - \rho_s(0, x)]^{BCS} \approx -\frac{4t}{\Delta(0, x)}\frac{\ln 2}{\pi}T \qquad (3)$$

Fermi liquid based theories introduce Landau parameters γ_{FL} which modify the low temperature slope:

$$[\rho_s(T) - \rho_s(0, x)]^{FL} \approx -\frac{4t}{\Delta(0, x)}\frac{\ln 2}{\pi}\gamma_{FL}T \qquad (4)$$

If it is presumed that $\rho_s(0) \propto T_c(x)$ and furthermore, that (unlike in the BCS case) $\rho_s(0)$ is independent of the hopping integral t, one arrives at a form in which $T_c(x)$ now appears explicitly.

$$\left[\frac{\rho_s(T)}{\rho_s(0, x)}\right]^{FL} \approx 1 - \gamma_{FL}\frac{t\nu}{\Delta(0, x)}\frac{4\ln 2}{\pi}\frac{T}{T_c(x)} \qquad (5)$$

In order for the right hand side of this equation to depend only on $T/T_c(x)$, γ_{FL} must counter the strong x dependence of $\Delta(0, x)$, or equivalently, T^*. Much of the literature has concentrated on x-dependences in the d-wave function shape [31,19,32] which are presumed to be responsible for some of the cancellation of x-dependences in the prefactor of $T/T_c(x)$ in Eq 5. We are inclined to view this as a peripheral effect since trends in the ρ_s data presented here are so strikingly systematic. Furthermore, a microscopic mechanism must be identified to suppress ρ_s to zero well below T^*. Strong thermal phase fluctuations have been suggested as a possibility.

The main frame of Figure 3 plots this Fermi liquid result for ρ_s when $\gamma_{FL} = 1$. Here to make the fitting easier, we chose

3

slightly smaller than conventional values of t: $4t \approx 160 meV$ (although this is of no physical consequence) and introduced the (x-independent) d-wave shape modifications discussed in the first paragraph of this section, so that at optimal doping, agreement with experiment is reasonable. It can be seen that because of the way the Uemura relation is enforced, the discrepancies between theory and experiment are less than for the strict BCS predictions. Choosing appropriate values of γ_{FL} as shown in the inset of Figure 4 leads to precisely fitted slopes and less discrepancy at all T, as plotted in the main frame of Figure 4. It should be noted from this Figure, however, that once Fermi liquid parameters are introduced to fit the low T slope, there are discrepancies in the region around T_c. That the calculated curves go to zero well above the measured T_c is presumably a consequence of the fact that phase fluctuations need to be invoked in this temperature regime.

Fermi-liquid based approaches, thus, obtain the measured universal slope for the rescaled curves by introducing [13] x-dependent Landau parameters. These approaches also presume a strong interconnection between the universality found in both the rescaled and off-set plots of $\rho_s(T)$ and the Uemura relation. There have been a variety of experimental studies [31,32] which build on this Fermi liquid picture. Indeed, this approach represents the most natural extrapolation of our BCS-based intuition. The theoretical work of Ioffe and Millis [19] suggests that these Landau parameters arise from coupling to phase fluctuations, and, in view of the systematic x-dependences inferred above, it might be reasonable to speculate that coupling to these phase fluctuations becomes progressively more important with underdoping.

C. Generalized Mean Field Theories of the Cuprate Pseudogap

There are two other approaches to pseudogap physics which are based on mean field theoretic approaches. In intrinsic models, T^* is taken to be the mean-field superconducting transition temperature, and either phase fluctuations (as in the Fermi liquid based theories, we have discussed above) or pair fluctuations (to be discussed below) suppress ρ_s to zero at a temperature well below T^*. In extrinsic theories T^* marks the onset of another order parameter. Some support for adopting a mean field theoretic approach comes from the fact that T^* and T_c can be orders of magnitude apart, so that it seems reasonable to establish an improved mean field theory, and then append fluctuation effects as they appear appropriate. In extrinsic models the pseudogap arises from physics other than superconductivity. Both extrinsic and intrinsic classes of mean field theories account for $T_c \neq T^*$ from the outset, and the behavior of ρ_s is at *low* temperatures already intimately connected to the x-dependent pseudogap physics responsible for the separation of the higher temperatures T_c and T^*, as will be discussed below.

In both the intrinsic or extrinsic mean field schools, the *superconducting pseudogap state* ($T \leq T_c$) is associated with the generalized equations for the gap and chemical potential μ
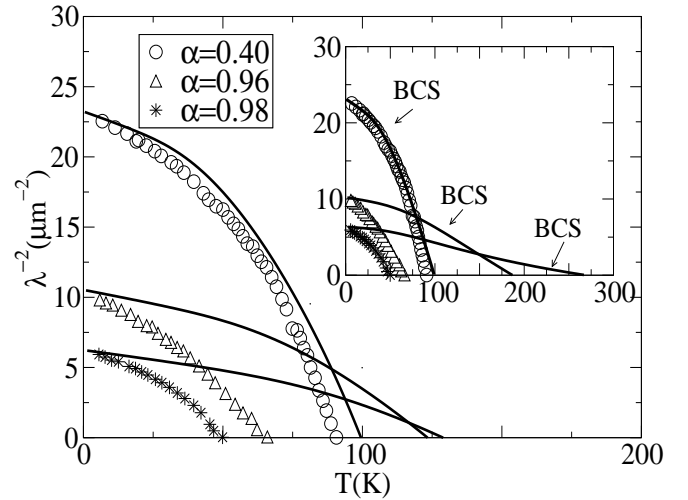
FIG. 3. Comparison between Fermi liquid based theories without Landau parameters and experiment for three values of x. Main figure: includes the Uemura constraint, as discussed in text. Upper right inset: theoretical prediction for strict BCS theory.
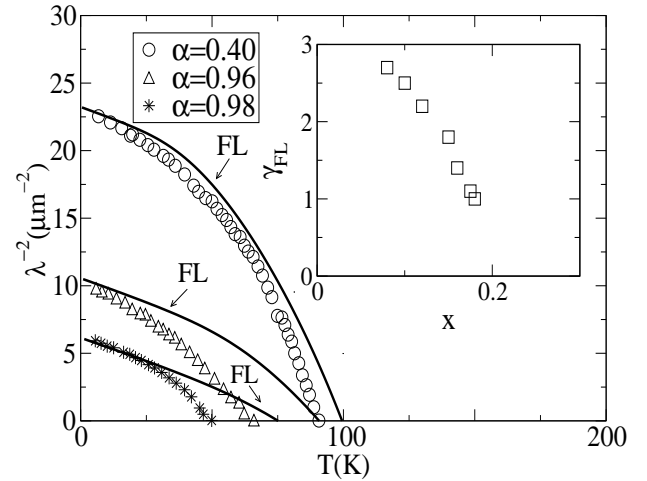
FIG. 4. Fermi liquid based fits to ρ_s for three values of x with Landau parameters inserted to fit the low T slopes. The appropriate values of Landau parameters for all seven samples of Figure 2 are shown in the inset.

$$g^{-1} + \sum_{\mathbf{k}} \frac{1 - 2f(E_{\mathbf{k}})}{2E_{\mathbf{k}}} \varphi_{\mathbf{k}}^2 = 0 \qquad (6)$$

$$n = 2 \sum_{\mathbf{k}} \left[f(E_{\mathbf{k}}) + v_{\mathbf{k}}^2 (1 - 2f(E_{\mathbf{k}})) \right] \qquad (7)$$

where n is the electron filling factor, $\varphi_{\mathbf{k}}$ represents a general d-wavefunction shape, and g the superconducting coupling constant. The quantity $v_{\mathbf{k}}$ is the coherence factor (described below) and $E_{\mathbf{k}}$ is the fermionic excitation energy which depends on the superconducting order parameter Δ_{sc} and the pseudogap energy scale Δ_{pg}. The extrinsic pseudogap is associated with the mixing of different $\mathbf{k}$ states, leading to the dispersion:

$$E_{\mathbf{k}}^{extrinsic} = \sqrt{(\epsilon_{\mathbf{k}}^{pg} - \mu)^2 + \Delta_{sc}^2(\mathbf{k})} \qquad (8)$$

4

where [8,7,6,33] $\epsilon_{\mathbf{k}}^{pg} = -\sqrt{\xi_{\mathbf{k}}^2 + \Delta_{pg}(\mathbf{k})^2}$

While there are a variety of different intrinsic theories of the normal state pseudogap [11,12,16] only the last of these [16] is readily compared with the extrinsic model discussed above, principally because it, too, represents a mean field treatment in which the two scales Δ_{sc} and Δ_{pg} appear. A detailed theoretical basis for this theory is given in a series of papers (See Ref. [16] and references therein). The principal assumption is a ground state wavefunction of the BCS form [34] with arbitrarily strong coupling constant g (which is parameterized via T^*/T_c or equivalently α) and self consistent μ so that Eqs. (6) and (7) apply.

Below T_c this strong coupling leads to pairing fluctuations of very low frequency and momentum contributing a fermion self-energy similar to the superconducting self-energy, leading to a more BCS-like dispersion: [16]

$$E_k^{intrinsic} = \sqrt{(\xi_{\mathbf{k}} - \mu)^2 + \Delta^2(\mathbf{k})} \qquad (9)$$

where $\Delta^2(\mathbf{k}) = \Delta_{pg}^2(\mathbf{k}) + \Delta_{sc}^2(\mathbf{k})$. We may now write the coherence factors and quasi-particle velocities in consolidated notation; these are given by $v_{\mathbf{k}}^2 = \frac{1}{2}(1 - (\epsilon_{\mathbf{k}} - \mu)/E_{\mathbf{k}})$, and $\partial_{\mathbf{k}}\epsilon_{\mathbf{k}}$, [33] with $\epsilon_{\mathbf{k}} = \xi_{\mathbf{k}}$ and $\epsilon_{\mathbf{k}}^{pg}$ for the intrinsic and extrinsic cases, respectively. Interestingly, a substantial body of evidence [1] for a purportedly *extrinsic* pseudogap comes from assuming this *intrinsic* dispersion [35].

The schematic T and x dependences of the various energy gaps in the two scenarios are sketched in Fig. 5. For the extrinsic case (upper panel) superconductivity forms on top of a pre-existing pseudogap in the normal-state dispersion, which first appears at T^* and is weakly T-dependent below T_c. For the intrinsic case (lower panel) T^* marks a gradual onset of the pseudogap associated with pairing fluctuations, which are to be differentiated from simple phase fluctuations around strict BCS (mean field) theory. Below T_c, these fluctuations are similar to free bosons, with the condensed fraction ($\propto \Delta_{sc}^2$) increasing at the expense of the uncondensed fraction ($\propto \Delta_{pg}^2$) until the fully condensed $T = 0$ ground state is reached [16]. In effect, the normal-state pseudogap evolves smoothly into the superconducting energy. As for the T-dependence of the crossover, the bosonic degrees of freedom behave, to leading order, as an ideal Bose gas, so the pseudogap portion of the total gap decreases as:

$$\Delta_{pg}^2(T) \approx \Delta^2(T_c)(T/T_c)^{3/2}, \ \ T \le T_c \qquad (10)$$

This description of the bosons below T_c is *required* [36] to maintain the form of the mean field defined by Eqs.(6) and (7). While Eqs. (6), (7), and (10) are the results of a previously discussed microscopic formalism [16], they motivate a phenomenology which requires as input only the measured values for $\Delta(0, x)$ and $T_c(x)$. The T dependence of the full gap $\Delta(T, x)$ can be expressed in terms of the BCS functional form [37] with "transition temperature" T^*, as is also consistent with experiment [26].

The insets to Figs. 5 indicate the x dependencies of the gaps at $T = 0$. In the intrinsic case, which has more BCS-like dispersion, the gap at $T = 0$ is much larger than expected from

the measured T_c. Hence, the slope of ρ_s at low T would diminish with underdoping if thermally excited fermionic quasiparticles were the only important excitation at low T.

For the extrinsic case, while the pseudogap persists to $T = 0$, the gap for superconducting excitations vanishes as T_c vanishes with underdoping. Thus, these theories would predict a growing slope in ρ_s at low T.
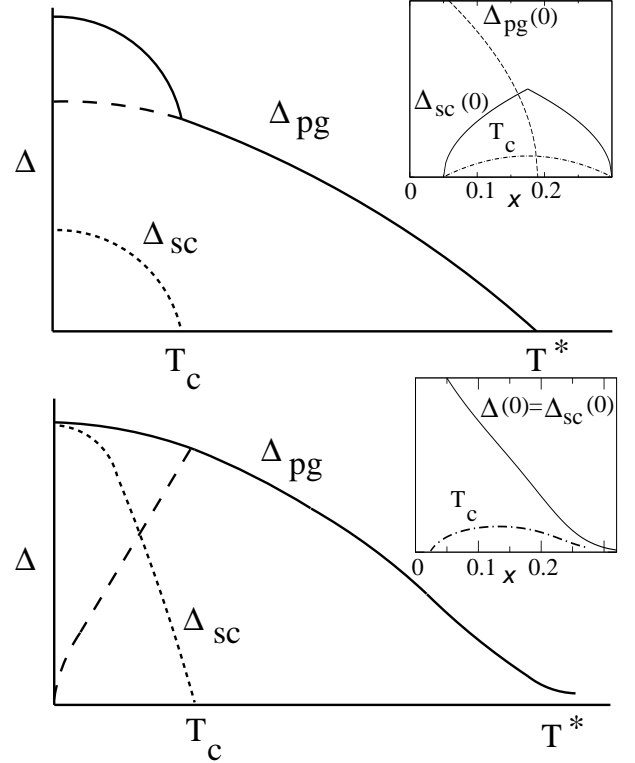

FIG. 5. Schematic energy gaps for extrinsic (upper panel) and intrinsic (lower panel) case. Dashed lines show Δ_{pg} below T_c, dotted lines are the superconducting order parameter while full lines represent $\sqrt{\Delta_{sc}^2 + \Delta_{pg}^2}$. Insets indicate the x dependence of the gaps at $T = 0$.

To further illustrate the important differences in dispersion between the two models, Fig. 6 presents a comparison [38] of the density of states at $T = 0$. In the intrinsic model there are two large gap-related peaks centered on the Fermi energy. Van Hove singularities are also apparent as relatively sharp structures. In contrast, there exist two pairs of peaks in the extrinsic theory. The larger pseudogap peaks are centered around $-\mu$, while the superconducting peaks appear around the Fermi energy. These differences will affect transport, and in particular $\rho_s(T)$.

For both extrinsic and intrinsic models ρ_s is of the form: [33]

$$\rho_{s_{ab}} = \sum_{\mathbf{k}} \frac{\Delta_{sc}^2}{E_{\mathbf{k}}^2} \left[\frac{1 - 2f(E_{\mathbf{k}})}{E_{\mathbf{k}}} + 2f'(E_{\mathbf{k}}) \right]$$

$$(\partial_a\epsilon_{\mathbf{k}}\partial_b\epsilon_{\mathbf{k}} - \frac{1}{2}\partial_a\epsilon_{\mathbf{k}}(\epsilon_{\mathbf{k}} - \mu)\partial_b\varphi_{\mathbf{k}}^2) \qquad (11)$$

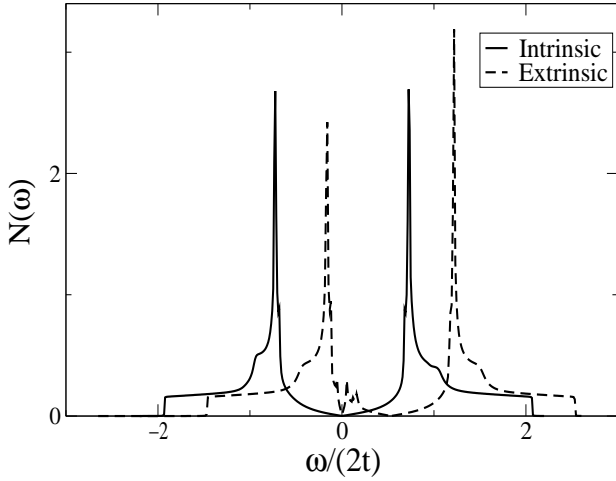For the extrinsic case [8,6,5], one finds at low T:

FIG. 6. Comparison of densities of states for intrinsic and extrinsic schools at $T = 0$ and $x = 0.07$. Modified d-wave gap shape is applied.

$$[\rho_s(T) - \rho_s(0)]^{extrinsic} \approx -\frac{4t}{\Delta_{sc}(0,x)}\frac{\ln 2}{\pi}T \quad (12)$$

showing that the low-T slope should grow as $t/\Delta_{sc}(0)$, ultimately diverging at the superconductor to insulator transition. Even if we consider normalized ρ_s, we find

$$\left[\frac{\rho_s(T)}{\rho_s(0)}\right]^{extrinsic} \approx 1 - \frac{\nu}{\Delta_{sc}(0,x)}\frac{4t\ln 2}{\pi}\frac{T}{T_c(x)} \quad (13)$$

and the same divergence. [39] Another consequence of the extrinsic dispersion is a $\sqrt{T}$ dependence [40] which sets in for $T/\mu \gg \Delta_{sc}^2/\Delta_{pg}^2$. This deviation from linearity is reflected in a convex shape for $\rho_s(T)$. These results (which were derived from the DDW scheme) appear to be consequences of the fermionic dispersion which is expected to be rather general for an extrinsic pseudogap.

In the intrinsic mean-field model, the bosons or uncondensed pairs couple in a natural way to the superfluid density. Above T_c (as in Hartree approximated GL theory) these pair fluctuations are responsible for the fermionic pseudogap which in turn depresses T_c relative to its mean field value, T^*. Within the superconducting phase, these uncondensed bosons contribute to the reduction of ρ_s at low T. Their contribution grows as the contribution of fermionic excitations decreases, as the pseudogap increases. The net result is very little apparent change in the low-T behavior of ρ_s with underdoping. Because of the BCS-like structure underlying the ground state (also consistent with the mean field equations (Eqs. (6) and (7)), the result for ρ_s is rather simple [16,41]

$$\rho_s^{intrinsic}(T) = [\Delta_{sc}^2(T)/\Delta^2(T)]\rho_s^{BCS}(\Delta(T)) \quad (14)$$

ρ_s^{BCS} is the BCS superfluid density that vanishes at T^*. Bosonic fluctuations simply rescale it by Δ_{sc}^2/Δ^2, which causes ρ_s to vanish at T_c. Bosonic degrees of freedom enter this equation in that they determine the T-dependence of Δ_{sc}. Rather than repeat the microscopic theory [16] presented elsewhere we introduce a more approximate but more accessible approach :

$$\Delta_{sc}^2(T) \approx \Delta^2(T)[1 - (\Delta^2(T_c)/\Delta^2(T))(T/T_c)^{3/2}], \quad (15)$$

which is applicable to the entire temperature regime. This leads to the following low T dependence of ρ_s

$$\left[\frac{\rho_s(T)}{\rho_s(0)}\right]^{intrinsic} = 1 - [A + B(T)]\frac{T}{T_c} \quad (16)$$

where $[A + B(T)]$ is weakly x dependent and

$$A = \frac{4\ln 2}{\pi}\frac{T_c}{\Delta(0,x)}\frac{t}{\rho_s(0)} \quad (17)$$

with

$$B(T) = \frac{\Delta^2(T_c)}{\Delta^2(T)}(T/T_c)^{1/2} \quad (18)$$

Here terms A and B correspond respectively to fermionic and bosonic excitations of the condensate. The quantity $t/\rho_s(0)$ must be be evaluated numerically; it is found to be of order unity, and independent of t. This fermionic term is precisely the same as in strict BCS theory and it reflects the full excitation gap Δ. Note that the fermionic contribution becomes progressively smaller as the insulator is approached. On the other hand, the bosonic contribution $B(T)$, which is only weakly T-dependent at low T, is insignificant in the overdoped regime where $\Delta(T_c)$ vanishes, and becomes progressively more important with underdoping. It can be seen that the x dependences in these two terms tend to oppose one another [16]. In effect, this rescaled equation (Eq.(16)) *represents a statement of the physical fact that the superfluid density is governed, even at low T by its ultimate vanishing at T_c, as follows from the dependence on the order parameter in Eq.(14)*. Here bosons are responsible for introducing the energy scale T_c in $\rho_s(T)$.
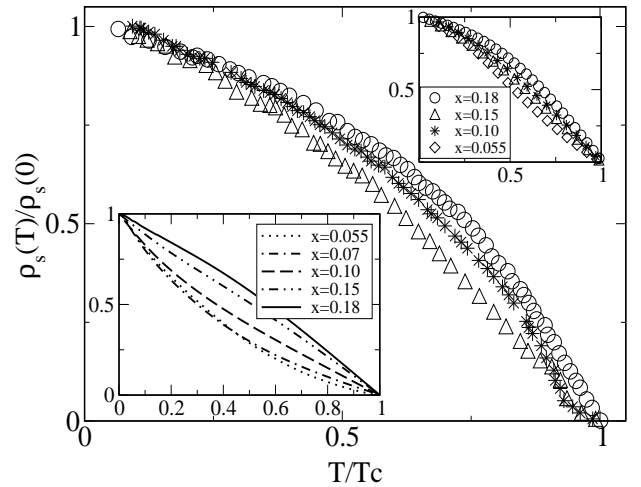

FIG. 7. Rescaled plot comparing experiment (main figure) with intrinsic (upper right) and extrinsic (lower left) mean field results.

To examine the universality in the off-set plot of the data we may write

$$[\rho_s(T) - \rho_s(0)]^{intrinsic} = -[A + B(T)]T \; [\rho_s(0,x)/T_c(x)]$$
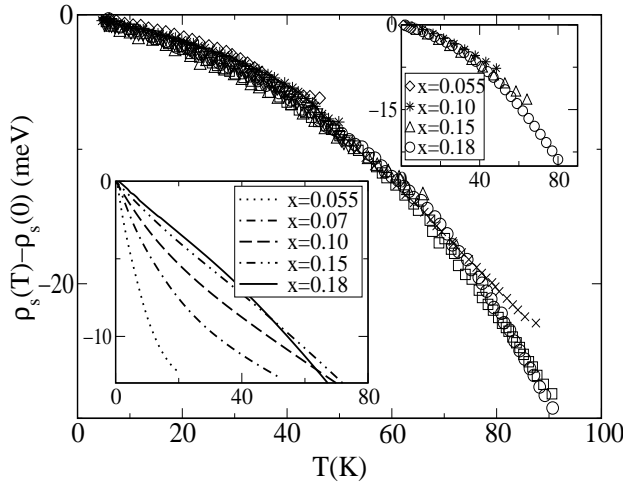$$\approx -[A + B(T)]T/\nu \quad (19)$$

FIG. 8. Offset plot comparing experiment (main figure) with intrinsic (upper right) and extrinsic (lower left) mean field results.

which shows that the off-set plots for $\rho_s(T)$ will be essentially x-independent only if the Uemura relation is then imposed. This will be consistent providing t is taken to be x-dependent, which, clearly, represents an over-simplified description of Mott insulating effects. Thus, within this mean field approach, Eq.(16) (which does not make any assumptions about $t(x)$), is a stronger and more general result than Eq.(19).

We turn now to detailed numerical studies of $\rho_s(T, x)$ in both extrinsic and intrinsic limits [38]. The resulting curves for $\rho_s(x, T)$ (which can be anticipated from low T expansions in Eqs. (12), (13), (16) and (19)) are plotted in Figures 7 and 8 for the rescaled and offset plots respectively. We can see from both Figures that for the extrinsic scenario [38] the general temperature dependence is progressively more convex with underdoping, as reflects the $\sqrt{T}$ dependence discussed above. This, in turn, is a consequence of the position of Δ_{pg} in the extrinsic dispersion. On the other hand, the intrinsic, or strong pairing attraction theory prediction leads to concave curves, which reflect both the general concavity of the strict BCS limit and the additional $T^{3/2}$ dependence [42] found in Eq.(10). For the extrinsic case, the effects of the divergent low T behavior discussed above and also noted elsewhere [39] are not particularly apparent in the rescaled plot of Figure 7. This may reflect that fact that for the parameters chosen here we were unable to satisfy the Uemura relation.

For the off-set plots in the right hand panels the extrinsic theory calculations illustrate the slope divergences and consequent lack of universality, anticipated by our analytic discussions. As can be seen, the intrinsic theory leads to quite universal curves, although here (in contrast to Figure 7) $t(x)$ was fitted to yield the experimental $\rho_s(0, x)$.

Because of the slope divergence and convex curvature at low x in the extrinsic approach, at this stage the comparison between theory and experiment (in Figures 7 and 8) favors an intrinsic origin for the pseudogap, as implemented either by the non-Fermi liquid mean field theory or by Fermi liquid based calculations with properly fitted Landau parameters. In-

deed, at the level of generalized boson-fermion microscopic models for Fermi liquid approaches [19], these two schemes may ultimately turn out to contain rather similar physics.

D. Conclusions

The experiments and theories addressed in this paper are at the heart of high T_c superconductivity, for they seek to unravel the nature of the excitations within the superconducting phase. To what extent are these excitations fermionic, bosonic or a mix of the two? We focus on $\rho_s(T)$, a uniquely superconducting quantity, because the experimental results are reasonably well established, [30,29] and the predictions of the two schools of theory differ significantly in their predictions for the evolution of the T-dependence of ρ_s with underdoping. The data exhibit a quasi-universal behavior in the low-T slope and curvature, that can be seen when $\rho_s(T)$ is plotted as $\rho_s(T)/\rho_s(0)$ vs T/T_c or as $\rho_s(T) - \rho_s(0)$ vs T. Thus, it seems that T_c is the only important energy scale, despite the large pseudogap in the normal state.

In the literature there are three simple ways of addressing the absence of the pseudogap energy scale in $\rho_s(T, x)$. 1) The pseudogap has an extrinsic origin, so the energy scale for fermionic excitations below T_c is [see Eqs. (12) and (13)] $\Delta_{sc}(x, T)$, which is proportional to T_c. 2) The pseudogap is a Fermi liquid [see Eqs. (4) and (5)], and Landau parameters γ_{FL} counter the x-dependence in the excitation gap or equivalently [27] in $T^*(x)$. Here the Uemura relationship is essential for introducing the scale $T_c(x)$. 3) The pseudogap is associated with a strong pair fluctuations that suppress T_c well below its mean field value, T^*. In this approach, bosonic (fluctuation) and fermionic single-particle excitations [see Eqs. (16) and (19)] contribute to the decrease of $\rho_s(T)$ at low T. Here the resulting expression for $\rho_s(T)$ involves the BCS contribution with a modified pre-factor reflecting the order parameter which necessarily vanishes at T_c: $\rho_s(T) = [\Delta_{sc}^2(T)/\Delta^2(T)]\rho_s^{BCS}(T)$.

It should be stressed that the intrinsic school pair fluctuation approach is probably the only well established or traditional route [23] to a fermionic pseudogap associated with homogeneous superconductivity. Nevertheless, by far the most widely applied intrinsic approach to the cuprate pseudogap is based on Fermi liquid theory. Much of the quantitative analysis within this scheme incorporates changes in the d-wavefunction shape with doping. In this way it is inferred that the Landau parameters are considerably less x-dependent than would be needed to cancel the x dependence in $T^*(x)$. We have not considered this possibility in this paper since our emphasis is on qualitative issues and since the striking universality reported here for seven different $YBa_2Cu_3O_{7-\delta}$ films makes it difficult to understand how changes in the shape of the d-wave gap could address this universality.

The essential contrast we have emphasized here is between two broad categories of pseudogap theories: *intrinsic* and *extrinsic* approaches. The Fermi liquid approach and pair-fluctuation scheme both belong to the former. These two

may be more directly interconnected, particularly in the Fermi liquid-based interpretation of Ref. [19]. Both start with an underlying fermion-boson Hamiltonian, where, in the Fermi liquid approach, the bosons are phase fluctuations phenomenologically coupled to the fermions, and to some extent the emphasis is more on Mott physics, and the associated softness of phase fluctuations. By contrast the pair-fluctuation scheme builds on the short coherence length and addresses the boson-fermion coupling at a fully microscopic level. This intrinsic mean field theory is a derivative of the traditional Gaussian fluctuation picture but with two modifications: calculations are done at the Hartree level (in order to include the fermionic pseudogap) and the attractive pairing interaction is taken to be arbitrary as is, therefore, T^*/T_c. Here, Mott physics enters in a more phenomenological way. Nevertheless, the bosons (which are not integrated out) are responsible for the shift in energy scale from $T^*(x)$ to $T_c(x)$.

While we have focussed on the D-Density wave model as a prototype for the extrinsic school, we believe it is reasonably generic and our conclusions are expected to apply to any CDW, stripe or alternative coexisting order parameter treated at a mean field level. For this reason our results should be of widespread interest to the community in large part because of the growing interest in quantum critical points. *Because of the vanishing of $\Delta_{sc}(x)$ as the insulator is approached, extrinsic theories are less compatible with the observed universality in $\rho_s(T)$.* Co-existing order parameters are found to lead to a breakdown of universality, and more generally to signatures within the superconducting phase which clearly distinguish under- and over-doped cuprates. These should be most visible near the superconductor insulator boundary. If future data on even more underdoped samples do not show significant deviations from the (thus far) "universal" curves for $\rho_s(x, T)$, we argue that this will provide substantial support for an intrinsic origin to the pseudogap in the context either of Fermi liquid-based or non-Fermi liquid based approaches to the superconductivity. Making a further distinction between these two should be possible through theoretical and experimental studies of the ac [43], as well as thermal conductivity, which studies are currently underway.

The discussion in this paper has focused mostly on mean field approaches. Other, non mean field approaches can be contemplated as well, but the observation of quasi-universality in $\rho_s(T, x)$ suggests that theories of the superconducting state should not deviate too strongly from simple BCS theory *at all T*. Additional support for this observation comes from the fact that there is often a very considerable separation (by orders of magnitude) between T^* and T_c which suggests that a carefully chosen mean field theory [10] is a more appropriate starting point for understanding the pseudogap phase than is an approach based on strict BCS physics with added fluctuation effects. One may view the off-set plot of the data in Figure 2 (lower left inset) as, one of the most interesting contributions of this paper and perhaps, the strongest constraint imposed by experiment on the nature of the excitations in the superconducting state. *This off-set plot suggests that the behavior of $\rho_s(T, x)$ at all T is a universal function and that there are no systematic differences between under-and optimally doped samples.* This, of course, is very surprising because the magnitude and temperature dependence of the excitation gap vary markedly for this range of x. Resolving this "paradox" has been a central issue of this paper.

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[36] The $T^{3/2}$ dependence of an ideal Bose gas enters because the pair susceptibility χ associated with Eq. (1), [which equation can be written as $1 + g\chi = 0$] necessarily has a q^2 dispersion, corresponding to a small q expansion of $G(k-q)G_o(k)$. For details see Ref. [16].

[37] A simple parameterization of the d-wave T dependence is $\Delta(T) \approx \Delta(0)\sqrt{1 - (T/T_c)^3}$.

[38] In the intrinsic case, the parameters were determined using the simple phenomenological approach, so that $T_c(x)$ and $\Delta(x,0)$ were fit to experiment [1]. For the extrinsic case gap and number equations from Ref. [44] were solved with $V_{DSC} = 0.8$ and $V_{DDW} = 2.0$. The modified d-wave gap shape (as in Figures 2 and 3) was used in both cases.

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